

The Laminar Boundary Layer in the Free Convection Dissolution of Salt

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ABSTRACT

An experimental investigation has been made to define the function of the boundary layer in the solution of salt by fresh water. The width of the boundary layer determined is an important factor and the relationship of the mass transfer of salt from the solid salt surface to the bulk fluid in a solution cavity is dependent upon conditions of the boundary layer. Evidence is presented to show the degree of mass transfer to be a function of the change in concentration throughout the boundary layer as well as a variation in concentration of the bulk fluid. Experimental data using photographic techniques show that maximum velocities of flow in the laminar free-convection boundary layer with mass transfer can be predicted from Eckert's integral method. Velocity and concentration profiles close to the solid surface should agree with those calculated by the integral method due to the agreement between calculated and experimental rates of mass transfer. However, the experimental velocities showed that the width of the velocity boundary layer was considerably greater than that calculated by the integral method assuming the concentration and velocity boundary layers to be of equal width.

INTRODUCTION

This study was undertaken as part of an investigation to develop a process for the solution mining of spherical salt cavities (1-4). A technique to provide the required geometry was developed in the laboratory using a blanket of gas or hydrocarbon liquid on top of the water in the cavity to control the washing process in the upper regions, and control of the water salinity regulated the development of the lower region. The water salinity was controlled by the water injection rate and the position of inlet and outlet tubing strings. Such a cavity is shown in Fig. 1.

The extension of laboratory techniques to field cavities required a good understanding of the mechanism of the dissolution of salt. It was shown that in laboratory cavities the salt dissolution was controlled by free convection boundary layer flow at the salt surface (5). The influence of forced convection resulting from the injection of fresh water and removal of the saline water could be neglected provided it was in the laminar mode. In either laboratory or field cavities, the rate of water injection would be such that turbulence would occur within the cavity only during the initial stages when the cavity is of small cross-sectional area. Hence, the injection rate would affect the dissolution only by establishing the salinity profile in the cavity which in turn determined the free convection dissolution. In laboratory cavities, the boundary layer would also generally be in the laminar mode. In large field cavities the dimensions are such that a turbulent boundary layer would generally occur even if the bulk fluid was in laminar flow, but the boundary layer would still control the dissolution process.

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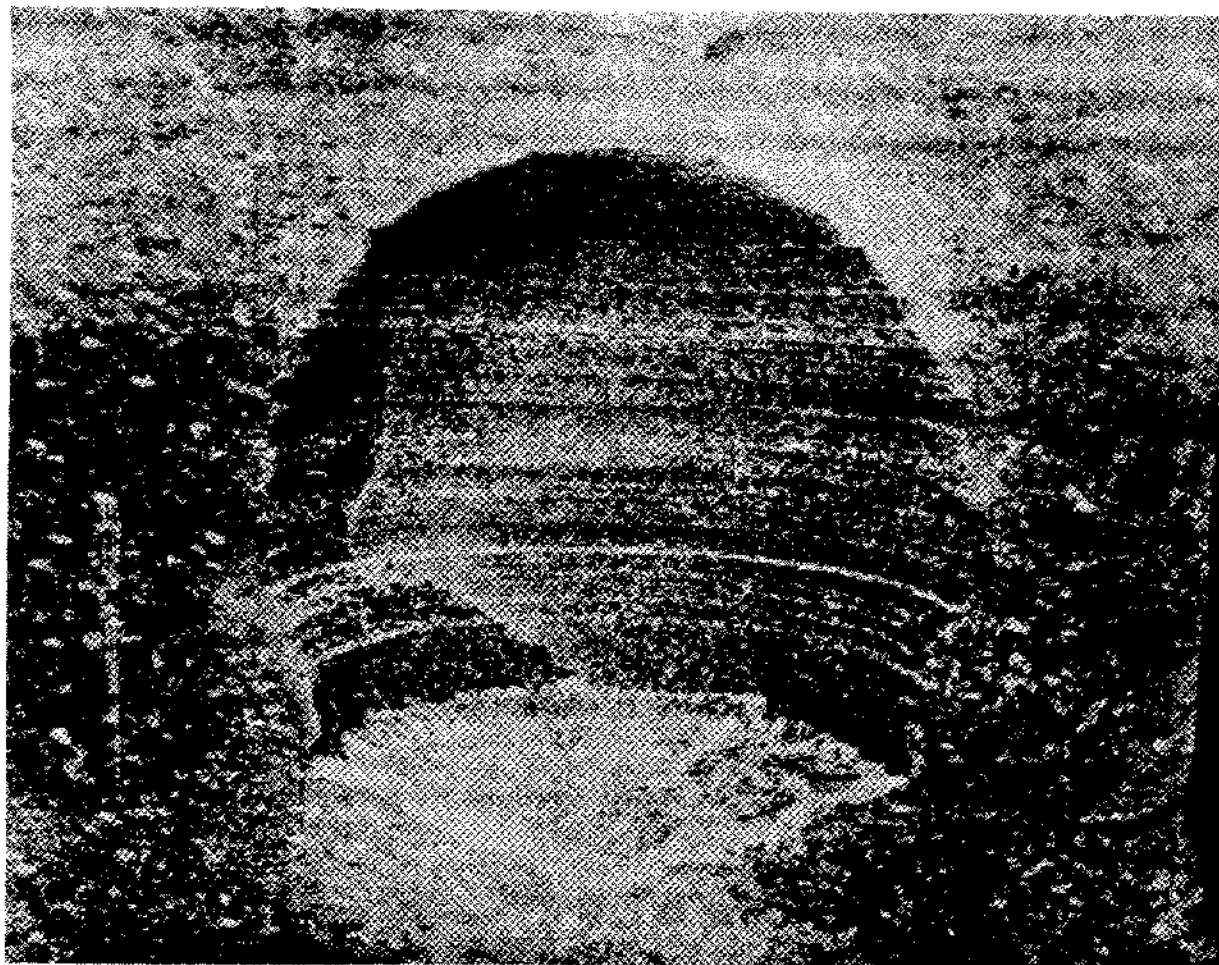


Figure 1. Laboratory Cavity.

The importance of free convection in the salt dissolution process led to a quantitative evaluation of the rate of dissolution with laminar free convection flow (5). An equation was developed to describe the dissolution that agreed very well with the experimental rates of free-convection salt transfer from a smooth vertical surface. The technique was taken from the work of Eckert (6) in the analogous heat transfer system and will be referred to as the integral method. The maximum velocity in the boundary layer, the width of the boundary layer, and the rate of salt removal so determined are described by the following equations:

$$U_m = 4.6 \left(\frac{DC_s x}{\nu} \right)^{1/2} \text{ cm/sec} \quad (1)$$

$$\delta = 1.6 \left(\frac{D\nu x}{C_s} \right)^{1/4} \text{ cm} \quad (2)$$

$$\left(\frac{dQ}{dt} \right)_x = 0.0732 \frac{C^{5/4} D^{3/4}}{\nu^{1/4} x^{1/4}} \text{ gm/(sq cm) (sec)} \quad (3)$$

This approach required the assumption that the width of the boundary layer of flow and the zone of concentration change would be the same, and this is applicable only with free convection in a gas. The concentration gradient at the salt surface was clearly in agreement with that predicted by the integral method because of the accurate prediction of mass transfer rates from the surface.

However, the boundary layer velocity could well be in error. Therefore, a study of the velocity profile in the boundary layer was undertaken.

THE VELOCITY PROFILE

For the analogous heat transfer in systems with large Prandtl numbers $\left(\frac{\rho C_p \nu}{k}\right)$, the velocity layer is wider than the temperature layer. Thus it can be expected that in mass transfer by free convection, the velocity layer should be wider than the concentration layer for systems where the analogous Schmidt number $\left(\frac{\nu}{D}\right)$ is large. From detailed analyses comparable to that originally undertaken by Pohlhausen (7), but using numerical techniques to obtain solutions, Ostrach (8), calculated a velocity boundary layer several times as wide as the temperature boundary layer for a Prandtl number $Pr = 1,000$. The water-salt system under consideration provides a Schmidt number (Sc) of approximately 1,000 which would, by analogy, indicate the expected velocity boundary layer to be considerably wider than the concentration boundary layer in the free-convection dissolution of salt.

The most important aspect of the boundary layer with respect to the problem of formation of salt cavities of controlled geometry is its influence on the rate of salt transfer from the surface. The velocity and concentration profile are of little practical concern, provided the mass transfer rates can be properly established. However, there has been little study of the free-convection boundary layer at high Schmidt numbers. Velocity profiles that have been measured generally are restricted to the heat transfer system and then usually for gases which have small Prandtl numbers. An investigation was undertaken to measure velocities in the boundary layer during the salt dissolution process to better understand the nature of the boundary-layer flow and to obtain information on the volume of water that can be transferred by the free-convection boundary layer.

PRELIMINARY INVESTIGATIONS

Preliminary investigations of a qualitative nature were carried out to find a technique that could be used to observe the boundary layer. It was apparent from visual examination that magnification would be required. Further, it was necessary to have particles in the boundary layer to provide reference points to observe the flow.

Of the many particles considered and tested to detail the flow, aluminum powder was found to be the most successful. It was found that the aluminum could be observed by reflected light when lighted from the side, and it showed up well as shadows when lighted from behind. Settling rates were so slow as to be negligible in relation to convection flow velocities, and these could be improved by allowing larger particles to settle out before initiating a quantitative investigation.

Dry aluminum powder could not be made to disperse in water so it was necessary to make the particles preferentially water wet before adding them to the water. A drop of nonionic detergent was used for this purpose.

The trace of the aluminum particles in the water gives a streak line which, in a steady-state system, is also the stream line. The rate of recession of the salt surface relative to the rate of flow within the boundary layer can be neglected, and hence this can be considered as a steady-state system. The particles therefore trace out stream lines and a qualitative measure of their velocities provides the velocity profile of the boundary layer.

The first idea investigated to provide visual enlargement of the boundary layer was to use an optical microscope as proposed by Schechter (9) to study velocity profiles in heat transfer. A standard slide projector was used to focus a narrow beam of light directly at the face of a salt block being dissolved by water within a rectangular transparent container. This could be observed in a darkened room from directly along the face of the salt with the microscope. The technique was not successful for quantitative measurements since observation through the microscope was difficult, flow velocities were too great, and thermal convection currents were induced by the light beam to mask the mass convection.

The second approach was to enlarge the image of the boundary layer by projection onto a screen. A lucite container 2.3 by 12.5 by 11.5 cm. was constructed from 1/16 Lucite sheet. This container could be inserted into the receptacle for the slide holder on a large slide projector. Small solid samples were placed in the container and dissolved with a solvent containing aluminum powder. The image projected on a suitable screen provided an enlarged view of the system. By proper adjustment, only the particles in the center of the container would be in focus to minimize the influence of the container walls. The projected boundary layer made it possible to observe in a qualitative manner the shape and dimensions of the boundary layer of flow, but the measurement of particle velocities could not be achieved with any reasonable degree of accuracy. However, a system that did give quantitative data was developed from the experience gained in these first attempts.

EXPERIMENTAL TECHNIQUE

The next approach used was direct close-up photography. A model to provide a visible free convection boundary layer was constructed from 1/16 inch sheet Lucite. The container had an upper section 15 centimeters wide by 2.5 centimeters deep by 12 centimeters high. The bottom was open to a reservoir with dimensions of 15 by 10 by 8 centimeters. The upper section provided a holder for the salt sample being dissolved, and the lower section was a fluid reservoir and a supporting base.

The boundary layer was provided by dissolving a salt sample cut from halite crystal blocks obtained from the Morton Salt Company mine at Grand Saline, Texas. The salt block could readily be cut to the required size for the container by placing a knife edge along a natural cleavage plane and then striking it with a light blow. The sample was then sealed on five sides with Hysol cement leaving one face exposed. It was placed vertically in the container at one extreme edge. The solvent was a saline solution containing aluminum powder to delineate the flow as previously discussed.

The photographic equipment was an Eastman Kodak Cine -- Special II 16 millimeter motion picture camera with an Ektar F 1.4, 25 millimeter lens on extension ring and Dupont 931A rapid reversal film. This equipment allowed a close-up picture such that a photographed element 1 centimeter high would fill one frame of the 16 millimeter film. A synchronous motor drive provided precise speed control at exactly 24 frames per second.

Two lighting techniques were successful. The first was to direct a photoflood light at a white background behind the container so that suspended particles would be outlined as shadows. The second, and most successful technique, was to use a black background with the aluminum particles illuminated by a light directed into the container from the side perpendicular to the salt face. The model was then filmed from directly in front along the salt face so that the boundary layer velocity profile could be seen.

A shutter setting of F 1.4 with a speed of 1/200 second provided the proper exposure with one photoflood lamp. This setting gave a very narrow depth of focus so that aluminum particles only in a narrow plane would give a distinct image. This plane was 1 centimeter from the camera lens and the lens was set back 2 millimeters from the container so that the particles in distinct focus were approximately 8 millimeters from the container walls.

It was found that successful pictures could be taken only if the salt was dissolved in quite saline water so that the boundary layer velocity was not excessive. With relatively fresh water, the particles were blurred and the boundary layer was narrow which made delineation more difficult.

Projection of the film with a 16 millimeter motion picture projector provided a qualitative picture of the flow comparable to that with the container in the slide projector as described before. Also, this system provided quantitative velocity data because particles could be traced from frame to frame and the velocity could be measured accurately. Figure 2 shows enlarged prints from a typical 16 millimeter film at 20 frame intervals. Individual particles can be clearly traced from frame to frame.



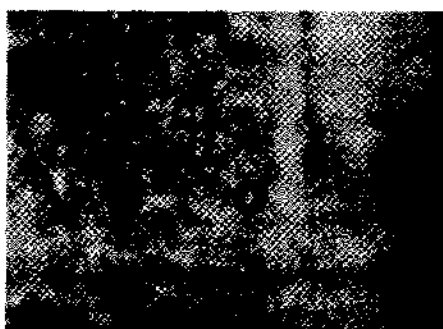
FRAME 11



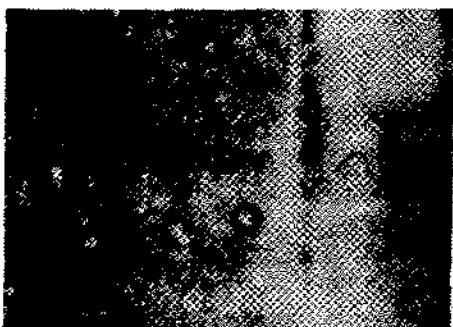
FRAME 31



FRAME 51



FRAME 71



FRAME 91

SCALE

1 centimeter



$C_s = .171 \text{ moles/liter}$

$x = 7.5 \text{ centimeters}$

Figure 2. Film Sequence for Particle Velocity.

A quantitative measure of the velocity of the particles was made with the aid of a microfilm viewer. A few frames taken with the camera focused on a standard centimeter scale provided a reference to find the magnification in the viewer relative to the actual model. A particular particle, distinctly in focus, would be chosen and its apparent distance from the salt face and the top of the frame measured with a vernier caliper. The film would then be advanced ten frames and the measurements repeated. The change in vertical height divided by ten frames gave the apparent velocity in cm/frame. The particle would be traced through several more intervals in this manner to confirm the measurements. The apparent distance from the salt surface would then be divided by the magnification to get the actual distance, and the actual velocity in cm/sec was found by dividing the apparent velocity by the magnification and multiplying by 24 frames per second.

RESULTS

Experimental particle velocities for three different values of solvent salinity are recorded on Fig. 3. The maximum experimental velocities and the width of the boundary layer are compared in Table I to the values calculated from Equations 1 and 2.

Very good agreement is observed between calculated and experimental maximum velocities. The experimental maximum velocity from run 2a is low, but Fig. 3 shows that there were probably no particles in the region of maximum flow, and hence the maximum velocity is probably higher than recorded. It was also observed that the calculated width of the boundary layer was in error by a factor of seven or eight times.

The concentration gradient at the salt surface is clearly in agreement with that predicted by the integral method because of the accurate prediction of mass transfer rates from the salt surface. The results suggest that the boundary layer very close to the salt behaves as predicted by the integral method, but the viscous nature of the brine causes the boundary layer to drag additional fluid with it so that the velocity boundary layer is much wider than predicted.

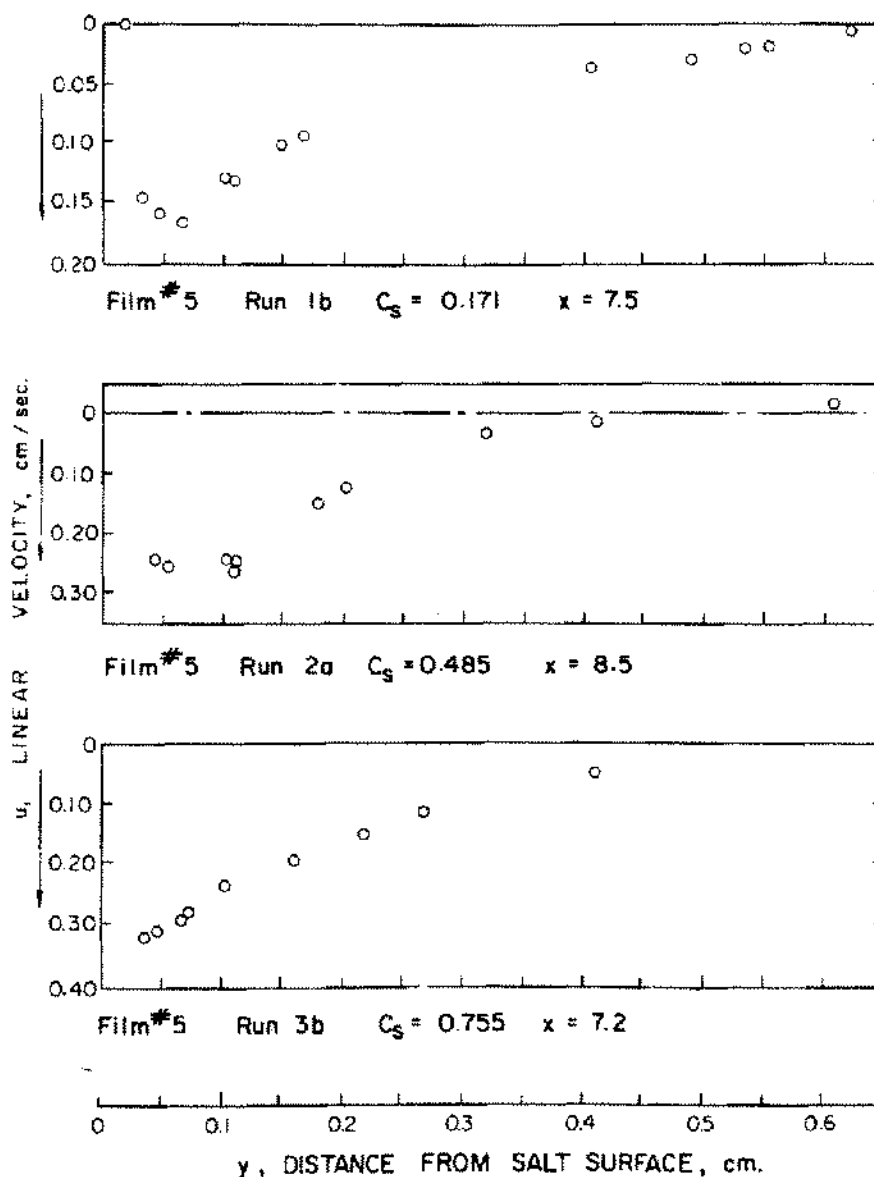


Figure 3. Boundary Layer Velocities.

TABLE I
COMPARISON OF CALCULATED AND EXPERIMENTAL U_m AND δ

Run	Calculated		Experimental	
	U_m cm/sec	δ cm	U_m cm/sec	δ cm
1b	0.16	0.092	0.17	0.63
2a	0.29	0.072	0.26	0.51
3b	0.34	0.062	0.33	0.50

Ostrach (8), by the so-called similarity transformation of boundary-layer theory, obtained a functional relationship in the heat transfer system between two dimensionless groups $\eta = \left(\frac{Gr_x}{y}\right)^{1/4} \frac{y}{x}$ and $F'(\eta) = \frac{ux}{2\nu_\infty(Gr_x)^{1/2}}$ for a constant Prandtl number. It appeared that a similar functional relationship would be valid for the problem of mass transfer by free convection. The Schmidt number (equivalent to the Prandtl number) varied between 1,000 and 1,030 for the three cases studied, so the variation was considered sufficiently small that one functional relationship between η and $F'(\eta)$ should apply for all cases.

Values of η and $F'(\eta)$ were calculated for the runs illustrated on Fig. 3 and plotted on Fig. 4. Very good agreement between the three sets of data is observed, although the results are not consistent with Ostrach's theoretical value for $Pr = 1,000$.

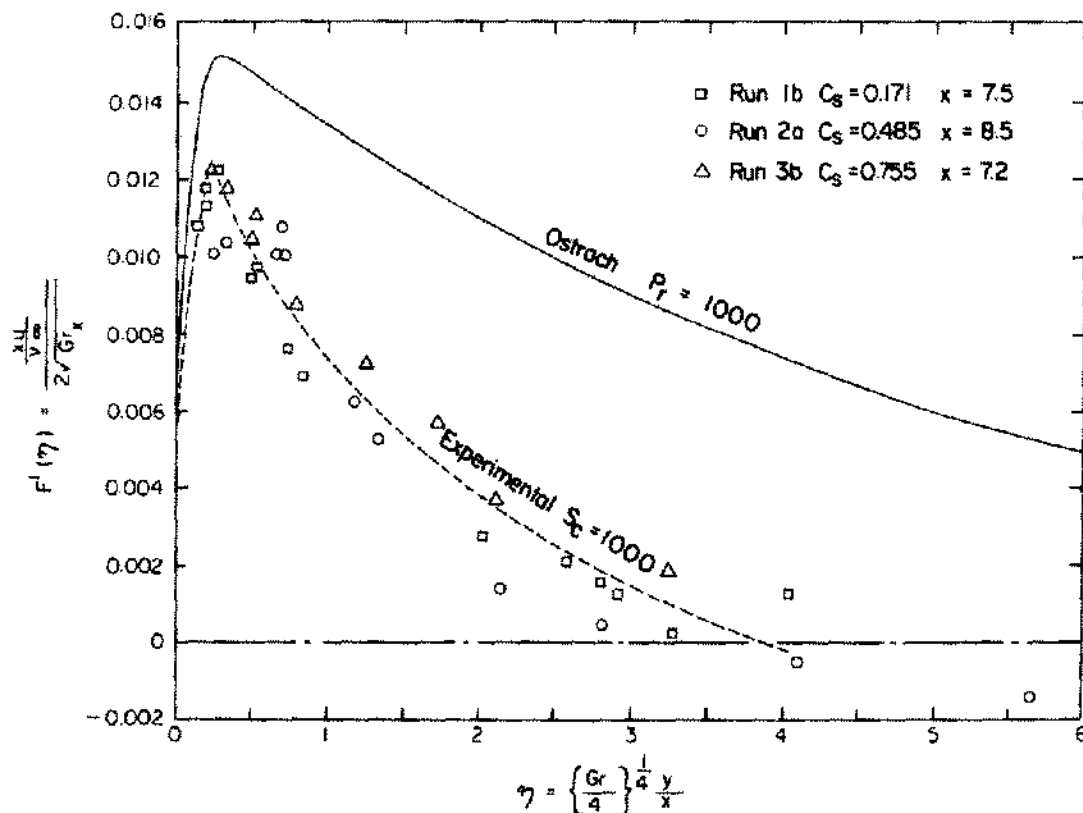


Figure 4. Functional Relationship Between η and $F'(\eta)$.

Of interest to this study was the determination of the fluid volume transferred in the boundary layer. The volume of flow is described by the integral:

$$V = \int_{y=0}^{\delta} u \, dy \quad \text{cm}^3/\text{sec} \quad (4)$$

where V is the total volume of water flowing in the boundary layer in an element of one centimeter width. From the definition of η and $F'(\eta)$ it follows that:

$$y = \frac{\eta x}{\left(\frac{Gr_x}{4}\right)^{1/4}} \quad \text{cm} \quad (5)$$

$$u = \frac{2 \nu_{\infty} (Gr_x)^{1/2} F'}{x} \quad \text{cm/sec} \quad (6)$$

Substitution of these values into Equation (4) yields:

$$V = 2.828 \nu_{\infty} (Gr_x)^{1/4} \int_{\eta=0}^{\left[\frac{(Gr)^{1/4}}{4} \frac{\delta}{x}\right]} F'(\eta) \, d\eta \quad \text{cm}^3/\text{sec} \quad (7)$$

The integral in Equation 7 can be evaluated for $Sc = 1,000$ by graphically integrating the experimental curve of $F'(\eta)$ versus η . This gives the equation:

$$V = 0.0525 \nu_{\infty} (Gr_x)^{1/4} \quad \text{cm}^3/\text{sec} \quad (8)$$

DISCUSSION OF RESULTS

The experimental relationship between $F'(\eta)$ and η has a shape similar to that predicted, and comparison of velocity profiles in terms of these dimensionless groups appears justified. However, the results are not consistent with Ostrach's theoretical value for $Pr = 1,000$ although the shape of the curve is similar to that predicted.

The difference between the experimental and theoretical results could be influenced by the boundary conditions. The model for the convection experiments had a width of approximately 12 centimeters. The downward flow through the 0.5 centimeter wide boundary layer was matched by an equal volume flowing upward through the other 12 centimeter section. This roughly approximated the condition in laboratory cavities. However, this did not provide an infinite y dimension and could have reduced the boundary layer width below that which has been achieved with a larger model. An infinite y dimension could give closer agreement between theoretical and experimental results, but the calculated volume of fluid transferred would be less representative of the volumes encountered in laboratory solution cavities.

Some sources of error are inherent in the apparatus. The boundary layer under investigation was approximately one centimeter back from the container walls, and some drag could have occurred. However, this drag could not have effected points of zero velocity and hence would have had no effect on the width of the boundary layer. The greatest effect would have been at the point of maximum velocity, and here the best agreement was obtained. Temperature effects from the light source could also have had some influence on convection, but tests with saturated solvent where all convection was by temperature alone suggested that this was not significant.

Ostrach was able to check his theory only with the heat convection experiments with gas. The prime assumption made in this analysis was that higher-order terms of ϵ [$\epsilon = \beta'(T_0 - T_{\infty})$] are negligible (8). This implies a coefficient of volumetric expansion β that is moderately small and a small temperature difference. In the analogous mass transfer system, β would include a term to convert from salinity to specific gravity as well as concentration difference C_s . It is not clear that higher-order terms of ϵ are negligible in this mass transfer problem.

It is hoped that this presentation may stimulate further interest in detailed study of the boundary layer velocities in the free convection mass transfer in infinite systems. The technique outlined could readily be applied to investigate systems of other dimensions, and thereby further check the results predicted by the Ostrach theory.

CONCLUSIONS

1. Experimental data using photographic techniques show that maximum velocities of flow in the laminar free-convection boundary layer can be predicted by the integral method. However, the experimental velocities showed that the width of the velocity boundary layer was considerably greater than that calculated by the integral method.
2. One functional relationship between η and $F'(\eta)$ applies for a range of values of C_s in the salt transfer problem under consideration.
3. The rate of fluid volume transfer in the boundary layer of flow can be obtained from the functional relationship.

LEGEND

C_p	Specific heat per unit mass
C	Salinity difference between salt surface and bulk fluid
D	Diffusivity Coefficient
$F'(\eta)$	$\frac{u x}{\nu_\infty} = \frac{1}{2(Gr_x)^{1/2}}$
F	$\int_0^\infty F'(\eta) d\eta$
Gr_x	Grashof number = $\frac{g\beta'\delta_w x^3}{\nu^2}$ or $\frac{g\beta C_s x^3}{\nu^2}$
k	Thermal conductivity
Pr	Prandtl number = $\frac{\rho C_p \nu}{k}$
Q	Mass of salt removed per unit area at a point on the surface
Sc	Schmidt number = $\frac{\nu}{D}$
T	Temperature
T_o	Temperature at solid surface
T_∞	Temperature of bulk fluid
U	Boundary layer velocity
U_m	Maximum velocity
x	Vertical distance from water surface
y	Horizontal distance from salt face
β'	Thermal expansion coefficient
β	Conversion from concentration difference to specific gravity difference
δ	Width of boundary layer
ϵ	Relative temperature difference = $\beta'(T_o - T_\infty)$
η	$\left(\frac{Gr_x}{4}\right)^{1/4} \frac{y}{x}$

ν	Kinematic viscosity
ν_{∞}	Kinematic viscosity of bulk fluid
ρ	Density

ACKNOWLEDGMENT

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